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Find the common ratio for the following sequence

Diagram illustrating three basic geometric sequences of the pattern 1(rn−1) up to 6 iterations deep. The first block is a unit block and the dashed line represents the infinite sum of the sequence, a number that it will forever approach but never touch: 2, 3/2, and 4/3 respectively. In mathematics, a geometric progression, also known as a geometric sequence, is a sequence of non-zero numbers where each term after the first is found by multiplying the previous one by a fixed, non-zero number called the common ratio. For example, the sequence 2, 6, 18, 54, ... is a geometric progression with common ratio 3. Similarly 10, 5, 2.5, 1.25, ... is a geometric sequence with common ratio 1/2. Examples of a geometric sequence are powers rk of a fixed non-zero number r, such as 2k and 3k. The general form of a geometric sequence is a , a r , a r 2 , a r 3 , a r 4 , … {\displaystyle a,\ ar,\ ar^{2},\ ar^{3},\ ar^{4},\ \ldots } where r ≠ 0 is the common ratio and a ≠ 0 is a scale factor, equal to the sequence's start value. The distinction between a progression and a series is that a progression is a sequence, whereas a series is a sum. Elementary properties The n-th term of a geometric sequence with initial value a = a1 and common ratio r is given by a n = a r n − 1 . {\displaystyle a_{n}=a\,r^{n-1}.} Such a geometric sequence also follows the recursive relation a n = r a n − 1 {\displaystyle a_{n}=r\,a_{n-1}} for every integer n ≥ 2. {\displaystyle n\geq 2.} Generally, to check whether a given sequence is geometric, one simply checks whether successive entries in the sequence all have the same ratio. The common ratio of a geometric sequence may be negative, resulting in an alternating sequence, with numbers alternating between positive and negative. For instance 1, −3, 9, −27, 81, −243, ... is a geometric sequence with common ratio −3. The behaviour of a geometric sequence depends on the value of the common ratio. If the common ratio is: positive, the terms will all be the same sign as the initial term. negative, the terms will alternate between positive and negative. greater than 1, there will be exponential growth towards positive or negative infinity (depending on the sign of the initial term). 1, the progression is a constant sequence. between −1 and 1 but not zero, there will be exponential decay towards zero (→ 0). −1, the absolute value of each term in the sequence is constant and terms alternate in sign. less than −1, for the absolute values there is exponential growth towards (unsigned) infinity, due to the alternating sign. Geometric sequences (with common ratio not equal to −1, 1 or 0) show exponential growth or exponential decay, as opposed to the linear growth (or decline) of an arithmetic progression such as 4, 15, 26, 37, 48, ... (with common difference 11). This result was taken by T.R. Malthus as the mathematical foundation of his Principle of Population. Note that the two kinds of progression are related: exponentiating each term of an arithmetic progression yields a geometric progression, while taking the logarithm of each term in a geometric progression with a positive common ratio yields an arithmetic progression. An interesting result of the definition of the geometric progression is that any three consecutive terms a, b and c will satisfy the following equation: b 2 = a c {\displaystyle b^{2}=ac} where b is considered to be the geometric mean between a and c. Geometric series This section may contain material unrelated or insufficiently related to its topic, which is the topic of another article, Geometric series. Please help improve this section or discuss this issue on the talk page. (February 2014) (Learn how and when to remove this template message) Main article: Geometric series 2 + 10 + 50 + 250 = 312 − (10 + 50 + 250 + 1250 = 5 × 312) 2 − 1250 = (1 − 5) × 312 Computation of the sum 2 + 10 + 50 + 250. The sequence is multiplied term by term by 5, and then subtracted from the original sequence. Two terms remain: the first term, a, and the term one beyond the last, or arm. The desired result, 312, is found by subtracting these two terms and dividing by 1 − 5. A geometric series is the sum of the numbers in a geometric progression. For example: 2 + 10 + 50 + 250 = 2 + 2 × 5 + 2 × 5 2 + 2 × 5 3 . {\displaystyle 2+10+50+250=2+2\times 5+2\times 5^{2}+2\times 5^{3}.} Letting a be the first term (here 2), n be the number of terms (here 4), and r be the constant that each term is multiplied by to get the next term (here 5), the sum is given by: a (1 − r n) 1 − r {\displaystyle {\frac {a(1-r^{n})}{1-r}}} In the example above, this gives: 2 + 10 + 50 + 250 = 2 (1 − 5 4) 1 − 5 = − 1248 − 4 = 312. {\displaystyle 2+10+50+250={\frac {2(1-5^{4})}{1-5}}={\frac {-1248}{-4}}=312.} The formula works for any real numbers a and r (except r = 1, which results in a division by zero). For example: − 2 π + 4 π 2 − 8 π 3 = − 2 π + (− 2 π) 2 + (− 2 π) 3 = − 2 π (1 − (− 2 π) 3) 1 − (− 2 π) = − 2 π (1 + 2 π 3) 1 + 2 π ≈ − 54.360768. {\displaystyle -2\pi +4\pi ^{2}-8\pi ^{3}=-2\pi +(-2\pi)^{2}+(-2\pi)^{3}={\frac {-2\pi (1-(-2\pi)^{3})}{1-(-2\pi)}}={\frac {-2\pi (1+2\pi ^{3})}{1+2\pi }}\approx -54.360768.} Since the derivation (below) does not depend on a and r being real, it holds for complex numbers as well. Derivation To derive this formula, first write a general geometric series as: ∑ k = 1 n a r k − 1 = a r 0 + a r 1 + a r 2 + a r 3 + ⋯ + a r n − 1 . {\displaystyle \sum _{k=1}^{n}ar^{k-1}=ar^{0}+ar^{1}+ar^{2}+ar^{3}+\cdots +ar^{n-1}.} We can find a simpler formula for this sum by multiplying both sides of the above equation by 1 − r, and we'll see that (1 − r) ∑ k = 1 n a r k − 1 = (1 − r) (a r 0 + a r 1 + a r 2 + a r 3 + ⋯ + a r n − 1) = a r 0 + a r 1 + a r 2 + a r 3 + ⋯ + a r n − 1 − a r 1 − a r 2 − a r 3 − ⋯ − a r n − 1 − a r n = a − a r n {\displaystyle {\begin{aligned}(1-r)\sum _{k=1}^{n}ar^{k-1}&=(1-r)(ar^{0}+ar^{1}+ar^{2}+ar^{3}+\cdots +ar^{n-1})\\&=ar^{0}+ar^{1}+ar^{2}+ar^{3}+\cdots +ar^{n-1}-ar^{1}-ar^{2}-ar^{3}-\cdots -ar^{n-1}-ar^{n}\\&=a-ar^{n}\end{aligned}}} since all the other terms cancel. If r ≠ 1, we can rearrange the above to get the convenient formula for a geometric series that computes the sum of n terms: ∑ k = 1 n a r k − 1 = a (1 − r n) 1 − r . {\displaystyle \sum _{k=1}^{n}ar^{k-1}={\frac {a(1-r^{n})}{1-r}}.} Related formulas If one were to begin the sum not from k=1, but from a different value, say m, then ∑ k = m n a r k = a (r m − r n + 1) 1 − r , {\displaystyle \sum _{k=m}^{n}ar^{k}={\frac {a(r^{m}-r^{n+1})}{1-r}},} provided r ≠ 1 {\displaystyle r\neq 1} . If r = 1 {\displaystyle r=1} then the sum is of just the constant a {\displaystyle a} and so equals a (n − m + 1) {\displaystyle a(n-m+1)} . Differentiating this formula with respect to r allows us to arrive at formulae for sums of the form G s (n , r) := ∑ k = 0 n k s r k . {\displaystyle G_{s}(n,r):=\sum _{k=0}^{n}k^{s}r^{k}.} For example: d d r ∑ k = 0 n r k = ∑ k = 1 n k r k − 1 = 1 − r n + 1 (1 − r) 2 − (n + 1) r n 1 − r . {\displaystyle {\frac {d}{dr}}\sum _{k=0}^{n}r^{k}=\sum _{k=1}^{n}kr^{k-1}={\frac {1-r^{n+1}}{(1-r)^{2}}}-{\frac {(n+1)r^{n}}{1-r}}.} For a geometric series containing only even powers of r multiply by 1 − r2 : (1 − r 2) ∑ k = 0 n a r 2 k = a − a r 2 n + 2 . {\displaystyle (1-r^{2})\sum _{k=0}^{n}ar^{2k}=a-ar^{2n+2}.} Then ∑ k = 0 n a r 2 k = a (1 − r 2 n + 2) 1 − r 2 . {\displaystyle \sum _{k=0}^{n}ar^{2k}={\frac {a(1-r^{2n+2})}{1-r^{2}}}.} Equivalently, take r2 as the common ratio and use the standard formulation. For a series with only odd powers of r (1 − r 2) ∑ k = 0 n a r 2 k + 1 = a r − a r 2 n + 3 {\displaystyle (1-r^{2})\sum _{k=0}^{n}ar^{2k+1}=ar-ar^{2n+3}} and ∑ k = 0 n a r 2 k + 1 = a r (1 − r 2 n + 2) 1 − r 2 . {\displaystyle \sum _{k=0}^{n}ar^{2k+1}={\frac {ar(1-r^{2n+2})}{1-r^{2}}}.} An exact formula for the generalized sum G s (n , r) {\displaystyle G_{s}(n,r)} when s ∈ N {\displaystyle s\in \mathbb {N} } is expanded by the Stirling numbers of the second kind as [1] G s (n , r) = ∑ j = 0 s { s j } x j d j d x j [1 − x n + 1 1 − x] . {\displaystyle G_{s}(n,r)=\sum _{j=0}^{s}\left\lbrace {s \atop j}\right\rbrace x^{j}{\frac {d^{j}}{dx^{j}}}\left[{\frac {1-x^{n+1}}{1-x}}\right].} Infinite geometric series This section may contain material unrelated or insufficiently related to its topic, which is the topic of another article, Geometric series. Please help improve this section or discuss this issue on the talk page. (February 2014) (Learn how and when to remove this template message) Main article: Geometric series An infinite geometric series is an infinite series whose successive terms have a common ratio. Such a series converges if and only if the absolute value of the common ratio is less than one (|r|